

2016 Oral Exam, Probability and Statistics

1. (Individual) Let X_λ be a Poisson random variable with parameter $\lambda > 0$:

$$\mathbb{P}[X_\lambda = k] = \frac{\lambda^k}{k!} e^{-\lambda}, \quad \forall k \in \{0, 1, 2, 3, \dots\}.$$

What is the limiting distribution of $\sqrt{X_\lambda} - \sqrt{\lambda}$ as $\lambda \rightarrow \infty$?

2. (Individual) Suppose that X_1, \dots, X_n are independent identically distributed random variables in L^1 . Define, for $1 \leq k \leq n$,

$$S_k = \sum_{j=1}^k X_j.$$

What is the conditional expectation of S_{n-1} given S_n ?

3. (Individual) Three players A, B, C play chess in turn. In each game, the winning rate is half-half. The one who wins twice in row (i.e., wins twice consecutively) will be the final winner. The game starts with A v.s. B.

What is the probability for A to be the final winner?

How about 5 players, i.e., there are players A, B, C, D, E.?